

Binomial Questions

- 5 (a) (i) Obtain the binomial expansion of $(1 - x)^{-1}$ up to and including the term in x^2 .
(2 marks)

(ii) Hence, or otherwise, show that

$$\frac{1}{3 - 2x} \approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$$

for small values of x .

(3 marks)

- (b) Obtain the binomial expansion of $\frac{1}{(1 - x)^2}$ up to and including the term in x^2 .
(2 marks)

- (c) Given that $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ can be written in the form $\frac{A}{(3 - 2x)} + \frac{B}{(1 - x)} + \frac{C}{(1 - x)^2}$,
find the values of A , B and C .
(5 marks)

- (d) Hence find the binomial expansion of $\frac{2x^2 - 3}{(3 - 2x)(1 - x)^2}$ up to and including the term
in x^2 .
(3 marks)

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- 2 (a) Obtain the binomial expansion of $(1 - x)^{-3}$ up to and including the term in x^2 .
(2 marks)

- (b) Hence obtain the binomial expansion of $\left(1 - \frac{5}{2}x\right)^{-3}$ up to and including the term
in x^2 .
(2 marks)

- (c) Find the range of values of x for which the binomial expansion of $\left(1 - \frac{5}{2}x\right)^{-3}$ would
be valid.
(2 marks)

- (d) Given that x is small, show that $\left(\frac{4}{2 - 5x}\right)^3 \approx a + bx + cx^2$, where a , b and c are
integers.
(2 marks)
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5 (a) Find the binomial expansion of $(1+x)^{\frac{1}{3}}$ up to the term in x^2 . (2 marks)

(b) (i) Show that $(8+3x)^{\frac{1}{3}} \approx 2 + \frac{1}{4}x - \frac{1}{32}x^2$ for small values of x . (3 marks)

(ii) Hence show that $\sqrt[3]{9} \approx \frac{599}{288}$. (2 marks)

2 (a) (i) Find the binomial expansion of $(1+x)^{-1}$ up to the term in x^3 . (2 marks)

(ii) Hence, or otherwise, obtain the binomial expansion of $\frac{1}{1+3x}$ up to the term in x^3 . (2 marks)

(b) Express $\frac{1+4x}{(1+x)(1+3x)}$ in partial fractions. (3 marks)

(c) (i) Find the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ up to the term in x^3 . (3 marks)

(ii) Find the range of values of x for which the binomial expansion of $\frac{1+4x}{(1+x)(1+3x)}$ is valid. (2 marks)

Binomial Answers

5(a)(i)	$(1-x)^{-1} = 1 + (-1)(-x) + \frac{(-1)(-2)}{2}(-x)^2$ $= 1 + x + x^2$	M1 A1	2	First two terms + kx^2
(ii)	$\frac{1}{(3-2x)} = \frac{1}{3} \left(1 - \frac{2}{3}x\right)^{-1}$ $\approx * \left(1 + \frac{2}{3}x + \left(\frac{2}{3}x\right)^2\right)$ $\approx \frac{1}{3} + \frac{2}{9}x + \frac{4}{27}x^2$	B1 M1	3	Or directly substitute into formula; M1 power of 3 M1 other coefficients (allow one error) A1 CAO AG convincingly obtained
(b)	$(1-x)^{-2} = 1 + (-2)(-x) + \frac{(-2)(-3)(-x)^2}{2}$ $= 1 + 2x + 3x^2$	M1 A1	2	First two terms + kx^2
5(c)	$2x^2 - 3 =$ $A(1-x)^2 + B(3-2x)(1-x) + C(3-2x)$ $x=1 \quad -1 = C \times 1 \quad x = \frac{3}{2} \quad \frac{3}{2} = A \times \frac{1}{4}$ $C = -1 \quad A = 6$ $x=0 \quad (-3 = 6 + 3B - 3)$ or other value \Rightarrow equation in A, B, C $B = -2$	M1 M1 A1 m1 A1	5	Or by equating coefficients M1 same A1 collect terms M1 equate coefficients A1 2 correct A1 3 correct Follow on A and C
(d)	$\frac{6}{3-2x} - \frac{2}{1-x} - \frac{1}{(1-x)^2}$ $\approx \frac{6}{3} \left(1 + \frac{2}{3}x + \frac{4}{9}x^2\right) - 2(1+x+x^2)$ $-(1+2x+3x^2) \approx -1 - \frac{8}{3}x - \frac{37}{9}x^2$	M1A1F A1	3	Follow on $A B C$ and expansions CAO
Total			15	

2(a)	$(1-x)^{-3} = 1 + (-3)(-x) + \frac{(-3)(-4)(-x)^2}{2}$ $= 1 + 3x + 6x^2$	M1 A1	2	$1 \pm 3x + x^2$ term
(b)	$\left(1 - \frac{5}{2}x\right)^{-3} = 1 + 3\left(\frac{5}{2}x\right) + 6\left(\frac{5}{2}x\right)^2$ $= 1 + \frac{15}{2}x + \frac{75}{2}x^2$	M1 A1	2	$x \rightarrow \frac{5}{2}x$, incl. $\left(\frac{5}{2}x\right)^2$ seen or implied (or start again) CAO OE
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(c)	$\left \frac{5}{2}x\right < 1 \quad x < \frac{2}{5}$	M1A1	2	Sight of $\frac{\pm 5}{2}$ or $\frac{\pm 2}{5}$
	$= 8\left(1 + \frac{15}{2}x + \frac{75}{2}x^2\right) = 8 + 60x + 300x^2$	M1		$k \times \text{their} \left(1 - \frac{5}{2}x\right)^{-3}$
(d)	<p>Alternatively, start again:</p> $8 \times \text{expression or } k \times \left(1 - 3\left(\pm \frac{5}{2}x\right)\right)$	(M1) (A1)	2	ft only on $8 \left(1 - \frac{5}{2}x\right)^{-3}$
	CAO			
Total			8	

5(a)	$(1+x)^{\frac{1}{3}} = 1 + \frac{1}{3}x + \frac{1}{3}\left(-\frac{2}{3}\right)\frac{1}{2}x^2$	M1 A1	2	$1 + \frac{1}{3}x + kx^2$
(b)(i)	$\sqrt[3]{8\left(1 + \frac{3}{8}x\right)^{\frac{1}{3}}}$ $= 2\left(1 + \frac{1}{3}\left(\frac{3}{8}x\right) - \frac{1}{9}\left(\frac{3}{8}x\right)^2\right)$ $= 2 + \frac{1}{4}x - \frac{1}{32}x^2$	B1 M1 A1	3	$8^{\frac{1}{3}}(1+kx)^{\frac{1}{3}}$ Replacing x with kx in answer to (a) For numerical expression which would evaluate to answer given
	<p>Alternative:</p> <p>B1 – all powers of 8 correct: $8^{\frac{1}{3}} 8^{-\frac{2}{3}} 8^{-\frac{5}{3}}$</p> <p>M1 – powers of $3x$ (condone $3x^2$)</p> $2 + \frac{1}{2}x - \frac{1}{9} \frac{1}{8^{\frac{5}{3}}} 9x^2$ <p>A1 – see some arithmetic processing</p>			
	<p style="text-align: center;">must see 9s in last term</p>			
(ii)	$x = \frac{1}{3}: \sqrt[3]{8+1} = 2 + \frac{1}{4} \times \frac{1}{3} - \frac{1}{32} \times \left(\frac{1}{3}\right)^2$ $\sqrt[3]{9} = \frac{576 + 24 - 1}{288} = \frac{599}{288}$	M1 A1	2	Using $x = \frac{1}{3}$ in given answer Any correct numerical expression = $\frac{599}{288}$
Total			7	

2(a)(i)	$(1+x)^{-1} = 1 + (-1)x + px^2 + qx^3$	M1		$p \neq 0, q \neq 0$
	$= 1 - x + x^2 - x^3$	A1	2	SC 1/2 for $= 1 - x + px^2$
(ii)	$(1+3x)^{-1} = 1 - 3x + (3x)^2 - (3x)^3$	M1		x replaced by $3x$ in candidate's (a)(i); condone missing brackets
	$= 1 - 3x + 9x^2 - 27x^3$	A1	2	CAO SC x^3 -term : $1 - 3x + \frac{3}{9}x^2$ 1/2
(b)	Alt (starting again) $(1+3x)^{-1} = 1 - (3x) +$			
	$\frac{(-1)(-2)(3x)^2}{2!} + \frac{(-1)(-2)(-3)(3x)^3}{3!}$	(M1)		condone missing brackets accept 2 for 2!, 3.2 for 3!
	$= 1 - 3x + 9x^2 - 27x^3$	(A1)	(2)	CAO
	$\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$	M1		correct partial fractions form, and multiplication by denominator
	$1 + 4x = A(1+3x) + B(1+x)$			
	$x = -1, x = -\frac{1}{3}$	m1		Use (any) two values of x to find A and B
	$A = \frac{3}{2}, B = -\frac{1}{2}$	A1	3	A and B both correct
(c)(i)	Alt: $\frac{1+4x}{(1+x)(1+3x)} = \frac{A}{1+x} + \frac{B}{1+3x}$	(M1)		correct partial fractions form, and multiplication by denominator
	$1 + 4x = A(1+3x) + B(1+x)$			
	$A + B = 1, 3A + B = 4$	(m1)		Set up and solve
	$A = \frac{3}{2}, B = -\frac{1}{2}$	(A1)	(3)	A and B both correct
	$\frac{1+4x}{(1+x)(1+3x)} = \frac{3}{2(1+x)} - \frac{1}{2(1+3x)}$	M1		
	$= \frac{3}{2}(1-x+x^2-x^3) - \frac{1}{2}(1-3x+9x^2-27x^3)$	m1		multiply candidate's expansions by A and B , and expand and simplify
	$= 1 - 3x^2 + 12x^3$	A1	3	CAO SC A and B interchanged, treat as miscopy. $(1 - 4x + 13x^2 - 40x^3)$
	Alt: $= \frac{1+4x}{(1+x)(1+3x)} = (1+4x)(1+x)^{-1}(1+3x)^{-1}$			
	$= (1+4x)(1-x+x^2-x^3)(1-3x+9x^2-27x^3)$	(M1)		write as product, using expansions condone missing brackets on $(1+4x)$ only
(ii)	$= 1 - 4x + 13x^2 - 40x^3 + 4x - 16x^2 + 52x^3$	(m1)		attempt to multiply the three expansions up to terms in x^3
	$= 1 - 3x^2 + 12x^3$	(A1)	(3)	CAO
	$ x < 1$ and $ 3x < 1$	M1		OE and nothing else incorrect
	$ x < \frac{1}{3}$ (0.33)	A1	2	OE Condone \leq
Total			12	